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C-Prime Anti Fuzzy bi-ideals in Boolean like Semi Rings

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Abstract

Zadeh have introduced the concept of Fuzzy set. The concept of Boolean like Semi-Rings were introduced by K.Venkatesh, B.V.N.Murthy and N.Amarnath. Kedukodi, Satyanarayana and Booth were introduced the idea of C-prime Fuzzy ideals of Near rings. After that R.Rajeswari, N.Meenakumari and R.Azhagumeena have created the idea of C-prime Fuzzy bi-ideals in Boolean like Semi-Ring. The aim of this paper is to introduce the concept of C-Prime Anti Fuzzy bi-ideals in Boolean like Semi rings. An Anti-Fuzzy bi-idealμ of Boolean like semi-ring R is called C-prime if for all $x, y \in R, \mu(xy) \geq \min\{\mu(x), \mu(y)\}$and discuss some of their related properties.

Keywords

Boolean like Semi-rings, fuzzy ideal, anti fuzzy bi-ideal, c-prime anti-fuzzy bi-ideal.

1. Introduction

The concept of Fuzzy was introduced by Zadeh. Fuzzy has a definite meaning, which can be made more precise only through further elaboration and specification-including a closer definition of the context in which the idea is used. The notion of Fuzzy bi-ideals in Boolean like semi rings were introduced by R. Rajeswari and N. Meenakumari. After that, R. Azhagumeena, R. Rajeswari and N. Meenakumari have introduced the concept of c-prime fuzzy bi-ideals in Boolean like semi rings. We extend this idea to c-prime anti fuzzy bi-ideals in Boolean like semi rings and discuss some of the theorems with an example.

2. Preliminaries

Definition: 2.1

A non empty R with two binary operation ' +' and ' **.**' is called a **near ring** if

i. $(R, +)$ is a group (not necessarily abelian)
ii. $(R, .)$ is a Semigroup
iii. $x.(y + z) = x.y + x.z$ for all $x, y, z \in R$

Definition: 2.2

A system (R,+, •) a **Boolean semi ring** if and only if the following properties hold

i. (R,+) is an additive (abelian) group (whose ' zero' will be denoted by ' 0')
ii. (R,•) is a semigroup of idempotents in the sense aa=a, for all a ∈ R.
iii. $(b + c) = ab + ac$ and
iv. $abc = bac$,for all $a, b, c \in R$

Definition: 2.3

A non empty set R together with two binary+ and • satisfying the following condition is called a **Boolean like semi-ring**.

i. (R,+) is an abelian group

ii. **(**R,•**)** is a semi group

iii. $a.(b+c) = a.b + a.c$ for all a,b,c∈ R

iv. $a + a = 0$ for all a in R

v. $a(a+b+ab) = ab$ for all a,b∈ R

Definition: 2.4

The zero symmetric part of N is$\{n \in N / \ n0 = 0\}$and is denoted by N_0.N is called **zero-symmetric** if $N = N_0$.

Definition: 2.5

A subgroup B of **(**N,+**)** is said to be a bi-ideal of N if BNB ∩ **(**BN**)** * B ⊆B**.** In the case of zero symmetric near ring a subgroup B of **(**N ,+**)** is a **Bi-ideal** BNB ⊆ B.

Definition: 2.6

A fuzzy set in a set M is a function μ **:**M→[0,1].

Notation: 2.7

We shall use the notation μ_t called a level subset of μ for $\{x \in \ M \ / \ (x) \ \geq \ t\}$ where $t \in$ [0,1].

Definition: 2.8

Suppose A and B are sets such that A⊆B. Define 3_A**:** B →[0,1] by $3_A(x) = 1$ if x ∈ A 3_A(x) = 0 if x ∉ A. Then 3_A is called as **characteristic function** of A .

Definition: 2.9

Let μ is a fuzzy normal divisor with respect to the addition means that μ satisfies

i. $(x - y) \geq \ min\{\mu(x)\ ,\mu(y)\}$

ii. $(y + x - y) \geq \mu(x), for\ all\ x, y \in M$

Definition 2.14

The mapping g: U→V such that $(x) \leq \ g(y)$ whenever $x \ \leq \ y$ is called **homomorphism** (or) an order homomorphism.

Definition 2.15

Let μ be a fuzzy set defined on R. Then μ is said to be a **fuzzy bi –ideal** of R if

i. $(x - y) \geq \ min\{\mu(x)\ ,\mu(y)\ \}\ , x\ , y\ \in\ R$

ii. $(xyz) \geq \ min\ \{\mu(x)\ ,\mu(z)\ \}$ for all $x, y, z\ \in\ R$

Definition: 2.16

Let μ be a fuzzy set defined on R**.** Then μ said to be**anti fuzzy bi-idea**l of R, if

1) $\mu(x - y) \ \leq \ max\{\ \mu(x), \mu(y)\ \}, x, y$ c R

2) $\mu(xyz) \leq \ max\{\ \mu(x), \mu(z)\ \}, x, y, z$ c R

3. C – Prime fuzzy bi – ideals

Definition 3.1

A fuzzy bi-ideal μ in a Boolean like semi- ring R is called c-prime if for all $x, y \in\ R$, $(xy) \geq\ min\{\ \mu(x)\ ,\mu(y)\ \}$.

Example 3.2

Let μ be a fuzzy bi-ideal defined on R by μ(0) = 0.9, μ(a) = 0.5 ,μ(b) = 0.6,μ(c) = 0.7 . Then μ is a c - prime anti fuzzy bi-ideal in R.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	a	b	a	0

·	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	0	b	b
c	0	a	b	c

Lemma 3.3

Let μ be a fuzzy subset of R. Then μ is an anti fuzzy bi-ideal of R if and only if

for all t ∈ [0,1] each level subset μ_t is a bi-ideal of R.

Proof:

Let μ be an anti fuzzy bi-ideal of R and let t ∈ [0,1] for any x,y $\in \mu_t$

We have $(x - y) \leq max\{\mu(x), \mu(y)\} \leq t$

And so $x - y \in \mu_t$

Now let $y \in \mu, x, z \in R$

Then $(xyz) \leq max\{\mu(x), \mu(z)\} \leq t$

$xyz \in \mu_t$ therefore μ_t is a bi-ideal of R.

Conversely, Assume that μ_t is a bi-ideal of R for every t ∈ [0,1].

If $(x_0 - y_0) > max\{\mu(x_0), \mu(y_0)\}$ for some x_0, $y_0 \in$ R,

Then by taking $t_0 = \frac{1}{2}[\mu(x_0 - y_0) + max\{\mu(x_0), \mu(y_0)\}]$

We have $(x_0 - y_0) > t_0$, $\mu(x_0) < t_0$ and $\mu(y_0) < t_0$

Hence $,_0 - y_0 \notin \mu_{t_0}$ $, x_0 \in \mu_{t_0,} y_0 \in \mu_{t_0}$

Which is a contradiction

Therefore $,(x - y) \leq max\{\mu(x), \mu(y)\}$ for all $x, y \in R$.

If the condition (ii) is not true ,

Then for a fixed $x, z \in R$, there exists y ∈ R such that $(xyz) > max\{\mu(x), \mu(z)\}$

Let $p_0 = \frac{1}{2}[\mu((r + a)s + rs) + \mu(a)]$

Then $xyz \notin \mu_{t0}$.This is a contradiction. Hence proved.

Theorem 3.4

Let μ be a fuzzy subset of R. Then μ is a c-prime anti fuzzy bi-ideal of R if and only if for all t ∈ [0,1] each level subset μ_t, is a c-prime bi-ideal of R.

Proof:

Suppose that μ be a c-prime anti fuzzy bi-ideal of R. Then μ is an anti fuzzy bi-ideal of R. By Lemma 3.3, μ_t is a bi-ideal of R.

Let x ,y∈ R such that $xy \in \mu_t$.

Then $(xy) \geq t$.

Since μ is a c-prime anti fuzzy bi-ideal of R, we have $(xy) \geq min\{ \mu(x) , \mu(y) \}$

Thus $min \{ (x) , \mu(y) \} \geq t \Rightarrow \mu(x) \geq t \; or \; \mu(y) \geq t$.

Thus $x \in \mu_t$or $y \in \mu_t$.

Conversely, Assume that μ is a c-prime bi-ideal of R for any t ∈ [0,1].

Then μ_t is a bi-ideal of R.

Again by Lemma 3.3, μ is an anti fuzzy bi-ideal of R.

Since μ_t is a c-prime bi-ideal of R and $x, y \in \mu$, we have $x \in \mu_t or \; y \in \mu_t$ which implies that $\mu(x) \geq t \; or \; \mu(y) \geq t$.

Thus $(xy) \geq min \{\mu(x) , \mu(y) \}$.

Hence μ is a c-prime anti fuzzy bi-ideal of R.

Lemma: 3.5

Let B be a non empty subset of R. Then B is a bi-ideal of R if and only if the characteristic function μ_B of B is an anti fuzzy bi-ideal of R.

Proof:

Let B be a bi-ideal of R.

For, $x, y \in B, x - y \in B$.

i) Let $x, y \in R$

(a) If x,y∈ B, then $\mu_B(x) = 1$ and $\mu_B(y) = 1$

Thus $\mu(x - y) = 1 = \max\{\mu_B(x), \mu_B(y)\}$

(b) If x∈ B and y ∉ B, then $\mu(x) = 1$ and $\mu_B(y) = 0$

Thus $\mu(x - y) = 0 \leq \max\{\mu_B(x), \mu_B(y)\}$

(c) If x ∉ B and y ∈ B, then $\mu(x) = 0$ and $\mu_B(y) = 1$

Thus $\mu(x - y) = 0 \leq \max\{\mu_B(x), \mu_B(y)\}$

(d) If x∉ B and y ∉ B , then $\mu_B(x) = 0$ and $\mu_B(y) = 0$

Thus $\mu(x - y) = 0 = \max\{\mu_B(x), \mu_B(y)\}$

ii) Let x,y,z∈ R

(a) If x,z∈ B, then $\mu_B(x) = 1$ and $\mu_B(z) = 1$

Thus $\mu(xyz) = 1 = \max\{\mu_B(x), \mu_B(z)\}$

(b) If x∈ B and z ∉ B, then $\mu(x) = 1$ and $\mu_B(z) = 0$

Thus $\mu(xyz) = 0 \leq \max\{\mu_B(x), \mu_B(z)\}$

(c) If x ∉ B and z ∈ B, then $\mu(x) = 0$ and $\mu_B(z) = 1$

Thus $\mu(xyz) = 0 \leq \max\{\mu_B(x), \mu_B(z)\}$

(d) If x∉ B and z ∉ B , then $\mu_B(x) = 0$ and $\mu_B(z) = 0$

Thus $\mu(xyz) = 0 = \max\{\mu_B(x), \mu_B(z)\}$ Conversely, suppose μ_B is an anti fuzzy bi-ideal of R.Then μ_B is two valued.

Hence B is a bi-ideal of R.

Theorem 3.6

Let B be a non empty subset of R. Then B is a c-prime bi-ideal of R if and only if μ_B is a c-prime anti fuzzy bi-ideal of R.

Proof:

Suppose that B is a c-prime bi-ideal of R. and μ_B is the characteristic function of B.Then by Lemma 3.5, μ_B is an anti fuzzy bi-ideal of R.

Let $x, y \in R$. If xy ∈ B, then μ(xy) = 1.
Since B is a c-prime bi-ideal of R&$xy \in B$, we have $x \in B$ or $y \in B$.
Thus μ_B(x)=1 or μ_B(y) =1 which implies that $\mu_B(xy) \geq min\{\mu_B(x), \mu_B(y)\}$.If $xy \notin B$, then $\mu_B(xy) = 0$.
Thus $\mu(xy) \geq min\{\mu_B(x), \mu_B(y)\}$
Conversely,
Assume that $\mu_B$is a c-prime anti fuzzy bi-ideal of R.Then μ_B is a anti fuzzy bi-ideal of R.
By lemma 3.5,B is a bi-ideal of R .Let $x, y \in R$ such that $xy \in B$. Then $\mu(xy) = 1$.
Since $\mu(xy) \geq min\{\mu_B(x), \mu_B(y)\}$,we have min{$\mu_B$(x),$\mu_B$(y)} = 1.
Thus μ_B(x) = 1 or μ_B(y) = 1.
Hence x ∈ B or y ∈ B.

Lemma 3.7

If B is a bi-ideal of R then for any t ∈(0,1), there exists an anti fuzzy bi-ideal μ of R such that μ_t = B.

Proof:

Let $\mu: R \to [0,1]$ be a fuzzy set defined by μ(x) $= \{\begin{smallmatrix} t \text{ if } x \in B \\ 0 \text{ } Otherwise \end{smallmatrix}$ for all $x \in R$ where $t \in (0,1)$
Then clearly μ_t = B
It is easy to prove that $\mu(x - y) \leq max\{\mu(x), \mu(y)\}$ for all x,y∈ R.
Assume that $\mu(xyz) > max\{\mu(x), \mu(z)\}$
We have $(xyz) = 0$& μ(y) = t
This is impossible because B is a bi-ideal of R which proves the theorem.

Theorem 3.8

Let B be a c-prime bi-ideal of R. For any t∈(0,1), there exists a c-prime anti fuzzy bi-ideal of R such that μ_t = B.

Proof:

Let t ∈ (0,1).Then by lemma 3.7, there exists an anti fuzzy bi-ideal μ of R defined by μ(x) = $\{\begin{smallmatrix} t \text{ if } x \in B \\ 0 \text{ } Otherwise \end{smallmatrix}$ such that μ_t = B.
Suppose μ is not a c-prime anti fuzzy bi-ideal of R. Then there exists $x, y \in R$ such that $(xy) \leq min\{\mu(x), \mu(y)\}$.
Using by definition of μ, we get $(x) = 0, \mu(y) = 0$ and $\mu(xy) = t$.
Thus we get $xy \in B$ & $x \notin B$ & $y \notin B$.This is a contradiction since B is a c-prime bi-ideal of R.
Hence μ is a c-prime anti fuzzy bi-ideal of R.

Theorem 3.9

Let μ be a fuzzy subset of R with $I(\mu) = \{1, \alpha\}$ where $\alpha \in [0,1)$. Then μ is a c-prime anti fuzzy bi-ideal of R if and only if $R_\mu = \{x \in R \mid (x) = \mu(0)\}$ is a c-prime anti fuzzy bi-ideal of R.

Proof:

Suppose μ is a c-prime anti fuzzy bi-ideal of R. Then μ is an anti fuzzy bi-ideal of R.
Let $, y \in R_\mu$
$\Rightarrow (x) = 0 \text{ } and \text{ } \mu(y) = 0$
Since μ is a c-prime anti fuzzy bi-ideal of R. Now$(xy) \geq min\{\mu(x), \mu(y)\}$

$= mi\{0,0\} = 0$

And so $(xy) = \mu(0)$

$mi\{ \mu(x), \mu(y)\} = 0$

$\Rightarrow (x) = 0 \; or \; \mu(y) = 0$

Thus $x \in R_\mu or \; y \in R_\mu$

$R_\mu$is a c-prime bi-ideal of R

Conversely,Assume that $R_\mu$is a c-prime bi-ideal of R.

Let $x, y \in \; , xy \in R_\mu$,we have $x \in R_\mu \; or \; y \in R_\mu$

$\Rightarrow (x) = 0 \; or \; \mu(y) = 0$

$(xy) \geq min\, \{\mu(x) \, , \mu(y)\}$

Hence μ is a c-prime anti fuzzy bi-ideal of R.

Theorem:3.10

Let $: U \rightarrow V$ is a homomorphism. If μ is c-prime bi-ideal of R, then$g^{-1}(\mu)$ is a c-prime anti fuzzy bi-ideal of R.

Proof:

Let $: U \rightarrow V$ is a homomorphism, γ be an anti fuzzy bi-ideal of R and μ be the

pre-image of a γ under f.

Then $\mu(x - y) = \gamma(\theta(x - y)) = \gamma(\theta(x) - \theta(y)) \leq$max$\{ \gamma(\theta(x) - \theta(y)\}$

$= \max\{\mu(x), \mu(y)\}$.

Now, μ(xyz) = $\gamma(\theta(xyz)) = \gamma(\theta(x), \theta(y), \theta(z)) \leq$max$\{ \gamma(\theta(x) - \theta(z))\}$

$= \max\{\mu(x), \mu(z)\}$ for all x,y,z ∈ R

4. References

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